

A Single Model of Interacting Dark Energy: Generalized Phantom Energy or Generalized Chaplygin Gas

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Abstract I present a model in which dark energy interacts with matter. The former is represented by a variable equation of state. It is shown that the phantom crossing takes place at zero redshift, moreover, stable scaling solution of the Friedmann equations is obtained. I show that dark energy is most probably be either generalized phantom energy or the generalized Chaplygin gas, while phantom energy is ruled out as a dark energy candidate.

Keywords Dark energy · Dark matter · Phantom energy · Chaplygin gas

1 Introduction

Numerous cosmological observations show that our universe is pervaded with a mysterious dark energy which constitutes more than seventy percent of the cosmic energy density [1–4]. The presence of dark energy is essential get a spatially flat universe as is observed today. Although dark energy is dominant in the universe, however we don't know its composition and origin. Theoretically there are several candidates that can account for dark energy including the cosmological constant, quintessence, Chaplygin gas and phantom energy etc., to name a few. However no single candidate among these can completely and satisfactorily meet the observational data. Another problem associated with it is that the exact form of the equation of state (EoS) of dark energy is yet to be discovered. In literature, several exotic EoS are proposed (see for example [5–10]). I in this paper will discuss the dynamics of dark energy with a recently proposed EoS that combines three forms of dark energy equations of state including generalized Chaplygin gas, phantom energy and its generalized form. Thus this single model presents unified cosmological model of analyzing dark energy dynamics.

From the cosmological observations, it is now clear that there are three major cosmic ingredients including dark matter, radiation and dark energy. It is naturally expected that these cosmic species may entertain some form of interaction at some stage in the evolution

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of the universe. This interaction is specifically important in the present cosmic setting when we observe that the energy densities of dark matter and dark energy are almost of the same orders of magnitude. Thus it excludes radiation from the interacting system. Recently there have been several cosmological models build up having dark energy of the forms mentioned above interacting with matter [11–24]. It is interesting to note that most of these models predict matter dominated scenario in the late evolution of the universe where dark energy decays into matter. This conclusion apparently avoids the imminent cosmic singularities like the big rip or the big crunch. Thus it naturally predicts a cyclic form of the evolution of the universe from matter domination to dark energy domination, back and forth.

In the present paper, I model my system by first assuming the spatially flat background spacetime. Further, I assume the spacetime to contain the only dark matter and dark energy and having exotic interaction. Both dark matter and dark energy are specified by their respective equations of state. I choose a specific form of the interaction term that is well motivated from the dimensional considerations. The exact form of the interaction term will be originally motivated from the theory of quantum gravity which is as yet in the process of construction by a number of theorists. That term contains a coupling parameter whose numerical value determines the strength of the interaction while its signature describes the direction of flow of the energy. I shall assume its numerical value to be between 0 and 1 while its signature to be positive (showing flow of energy from dark energy to matter). Finally I perform stability analysis of dynamical system and discuss my results.

2 Modeling of Dynamical System

I start by assuming the background to be spatially flat homogeneous and isotropic Friedmann-Robertson-Walker (FRW) spacetime

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]. \tag{1}$$

The equations of motion corresponding to FRW spacetime filled with the two component fluid (matter and dark energy) are

$$\dot{H} = -\frac{\kappa^2}{6}(p_{de} + \rho_{de} + \rho_m), \tag{2}$$

$$H^2 = \frac{\kappa^2}{3}(\rho_{de} + \rho_m). \tag{3}$$

Here $\kappa^2 = 8\pi G$ is the Einstein’s gravitational constant and $H = H(t) \equiv \dot{a}/a$ is the Hubble parameter. Also subscripts ‘*m*’ and ‘*de*’ correspond to matter and dark energy respectively while *p* and ρ are respectively pressure and energy density. In (2), matter is assumed to be pressureless ($p_m = 0$) while the dark energy’s equation of state is considered of the form [25]

$$p_{de} = -Br^{2(\alpha-1)}\rho_{de}^\alpha, \quad \alpha \neq 0, \tag{4}$$

where *B* is a positive constant and α can be either positive or negative. Note that $\alpha = 1$ represents phantom energy, $\alpha > 0$ gives generalized phantom energy while $\alpha < 0$ represents generalized Chaplygin gas. Thus the generalized phantom energy and Chaplygin gas models can be unified by the above equation of state.

The energy conservation equation for the dynamical system under consideration is

$$\dot{\rho}_{de} + \dot{\rho}_m + 3H(\rho_m + \rho_{de} + p_{de}) = 0. \tag{5}$$

Due to interaction between the two components, the energy conservation would not hold for the individual components, therefore the above conservation equation will break into two non-conserving equations:

$$\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = -Q, \quad (6)$$

$$\dot{\rho}_m + 3H\rho_m = Q. \quad (7)$$

Here Q is the interaction term which in general is a function of energy densities and the Hubble parameter i.e. $Q(H\rho_m, H\rho_{de})$ which upon Taylor expansion yields the form given below in (8). This interaction is demanded to be positive to avoid the violation of the laws of thermodynamics [26]. I also insert a dimensionless coupling parameter c in Q to determine the strength of the interaction, thus I have

$$Q = 3Hc(\rho_{de} + \rho_m). \quad (8)$$

To study the dynamics of our system, I proceed by setting

$$x = \ln a = -\ln(1+z), \quad (9)$$

which is termed as the e-folding time parameter and z is the redshift parameter. Moreover, the density and pressure of dark energy can be expressed by dimensionless variables u and v as

$$u = \Omega_{de} = \frac{\rho_{de}}{\rho_{cr}} = \frac{\kappa^2 \rho_{de}}{3H^2}, \quad v = \frac{\kappa^2 p_{de}}{3H^2}. \quad (10)$$

The EoS parameter ω_{de} is conventionally defined as

$$\omega_{de} \equiv \frac{p_{de}}{\rho_{de}}, \quad (11)$$

which after using (10), becomes

$$\omega_{de} = \frac{v}{u}. \quad (12)$$

The density parameters of dark energy and dark matter are related as

$$\Omega_m = \frac{\kappa^2 \rho_m}{3H^2} = 1 - \Omega_{de} = 1 - u. \quad (13)$$

Using (8)–(13) in (6) and (7), I obtain the following autonomous system

$$\frac{du}{dx} = -3c - 3v + 3uv, \quad (14)$$

$$\frac{dv}{dx} = -3\alpha v \left(1 + \frac{v}{u} + \frac{c}{u}\right) + 3v(v+1). \quad (15)$$

Note that the above dynamical system matches with [28] if $\alpha < 0$. Their it was shown that interacting GCG could explain late time acceleration. I am here particularly interested in the cases when $\alpha = 1$ and $\alpha > 0$. The stability of this system is performed by first finding its

critical points i.e. the points for which the left hand sides of the system vanish identically. Thus I obtain the only critical point:

$$u_c = 1 - c, \tag{16}$$

$$v_c = -1. \tag{17}$$

It needs to be checked whether the system is stable about (u_c, v_c) . For this purpose, I linearize the system by substituting $u = u_c + \delta u$, and $v = v_c + \delta v$, where δu and δv are small perturbations about the critical point. I finally obtain

$$\frac{d\delta u}{dx} = 3v_c\delta u + 3(-1 + u_c)\delta v, \tag{18}$$

$$\frac{d\delta v}{dx} = \frac{3}{u_c^2}[\alpha(v_c^2 + cv_c)\delta u + [(1 + 2v_c - \alpha)u_c^2 - 2\alpha u_c v_c - \alpha c u_c]\delta v]. \tag{19}$$

Now the given dynamical system is stable about the critical point if the real parts of the eigenvalues are both negative. In this case, the critical point is called a stable node. However, if one is positive then such a point is termed a saddle point. If both positive, it is called an unstable node. The eigenvalues are

$$\lambda_1 = -3 - \frac{-3\alpha + 3\sqrt{\alpha(\alpha - 4c(1 - c))}}{2(1 - c)}, \tag{20}$$

$$\lambda_2 = -3 - \frac{-3\alpha - 3\sqrt{\alpha(\alpha - 4c(1 - c))}}{2(1 - c)}. \tag{21}$$

It can be shown easily that the real parts of both the eigenvalues are negative. In Figs. 1 and 2, I show phase space diagrams of the solutions of my dynamical system for different choices

Fig. 1 (Color online) The phase diagram of the interacting dark energy model with $c = 0.7$ and $\alpha = 0.07$. The curves correspond to the initial conditions $u(-2) = 1.2, v(-2) = -0.2$ (green); $u(-2) = 1.3, v(-2) = -0.3$ (blue); $u(-2) = 1.4, v(-2) = -0.4$ (red); $u(-2) = 1.5, v(-2) = -0.5$ (brown)

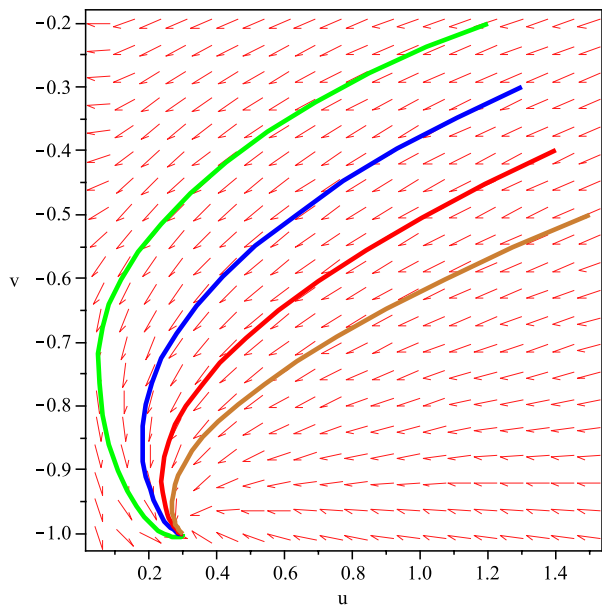


Fig. 2 (Color online) The phase diagram of the interacting dark energy model with $c = 0.8$ and $\alpha = 1$. The curves correspond to the initial conditions as given in Fig. 1

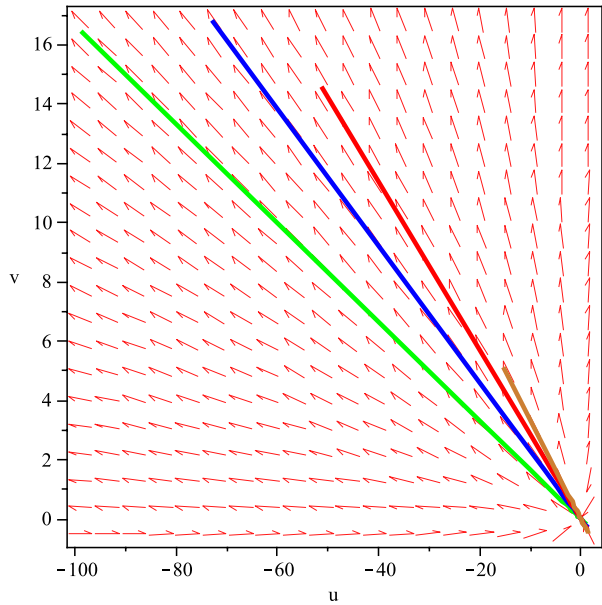
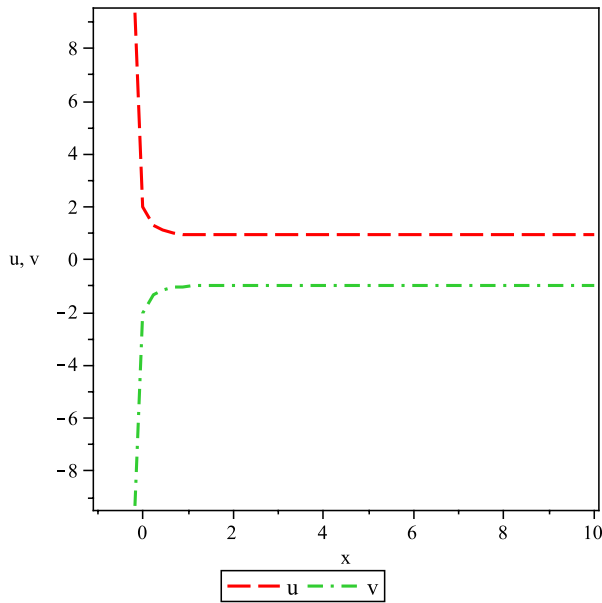


Fig. 3 The parametric functions u and v are plotted against the e-folding time parameter x . The parameters are fixed as $c = 0.7$ and $\alpha = 0.07$. The initial condition is $u(0) = 2, v(0) = -2$



of initial conditions. Here I took $\alpha = 0.07$ and $\alpha = 1$, representing generalized phantom energy and the phantom energy, respectively. Note that I am not discussing the case of $\alpha < 0$ as it already has been discussed in [28]. In Figs. 3 and 4, I provide pictorial representation of the solutions against the e-folding time parameter. It is interesting to note that the solutions are stable (without fluctuations or oscillations of any kind). Figure 5 shows an unstable

Fig. 4 The parametric functions u and v are plotted against the e-folding time parameter x . The parameters are fixed as $c = 0.06$, $\alpha = 1$. The initial condition is $u(0) = 2, v(0) = -2$

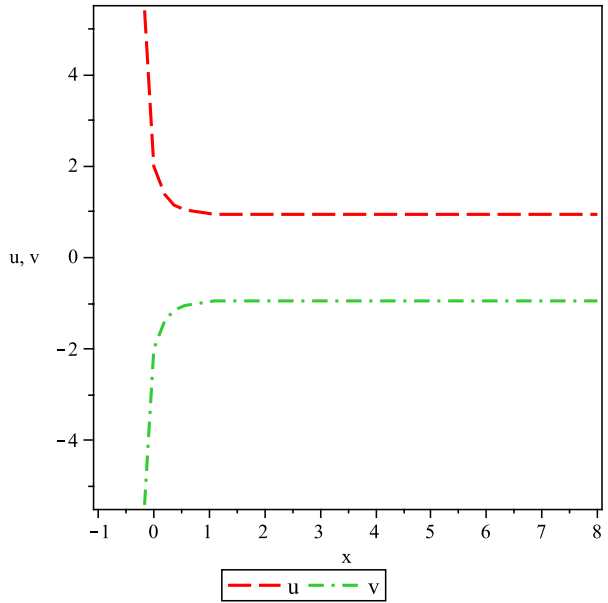
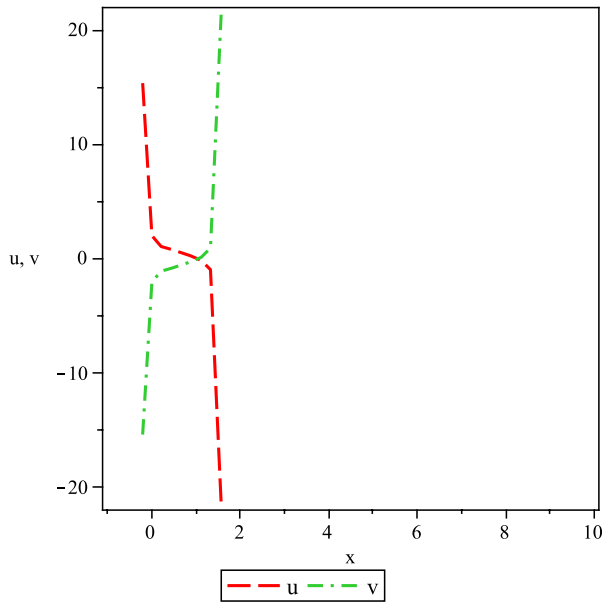


Fig. 5 The parametric functions u and v are plotted against the e-folding time parameter x . The parameters are fixed as $c = 0.6$, $\alpha = 1$. The initial condition is $u(0) = 2, v(0) = -2$



solution which is consequently of not much physical interest. However it puts an upper limit on the choice of $c = 0.6$ if the dark energy is of phantom type $\alpha = 1$. In Figs. 6 and 7, I plot the dark energy state parameter against e-folding time parameter x . The former one presents a smooth transition from the quintessence to phantom regime while this transition takes place at $x = 0$, which corresponds to the present epoch. The later figure shows a rather

Fig. 6 The EoS parameter $\omega_X = v/u$ is plotted against x . The parameters are fixed as $c = 0.7$ and $\alpha = 0.07$. The initial conditions are $u(0) = 1$, $v(0) = -1$ (solid line); $u(0) = 1.1$, $v(0) = -1.1$ (dots); $u(0) = 1.2$, $v(0) = -1.2$ (dashes); $u(0) = 1.3$, $v(0) = -1.3$ (dash dot)

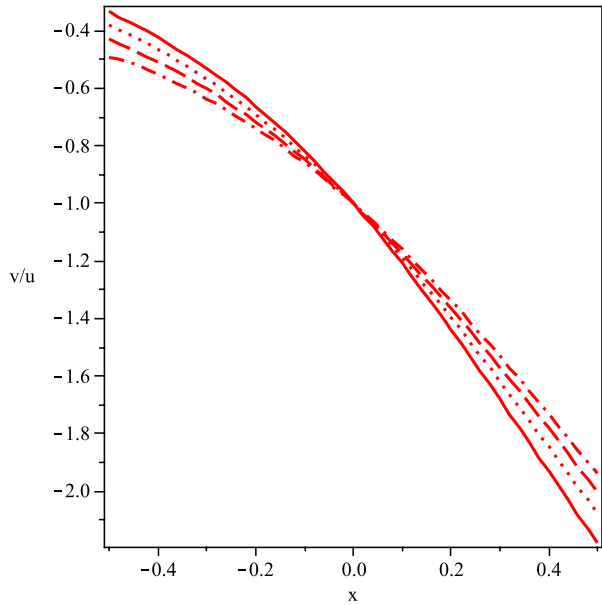
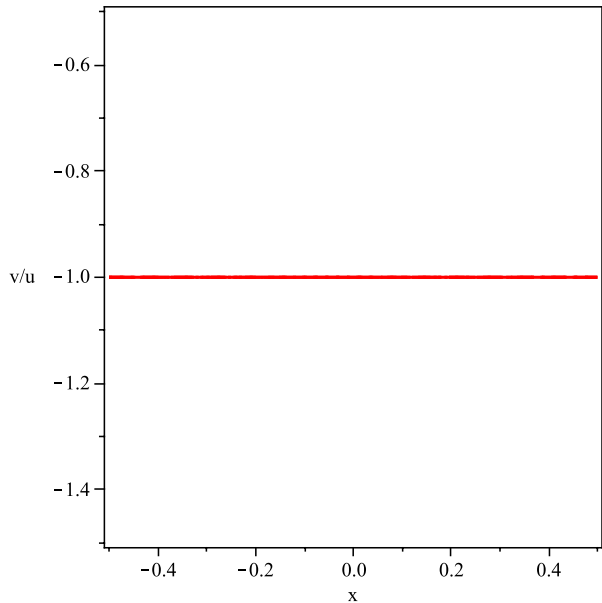


Fig. 7 The EoS parameter $\omega_X = v/u$ is plotted against x . The parameters are fixed as $c = 0.5$ and $\alpha = 1$. The initial conditions are the same as in Fig. 6



stale case where the state parameter remains unchanged throughout the variation of x . I also point out here that this last result is unaffected for various values of c . Hence it can be said that dark energy can most probably be either generalized Chaplygin gas or the generalized phantom energy.

3 Conclusion

Models of interacting dark energy possess tremendous potential to alleviate several cosmological puzzles, most notably the cosmic coincidence problem and the phantom crossing scenario. The model predicts a dynamical nature of dark energy where its equation of state and energy density both vary over cosmic time which turns out to be consistent with the observations. As seen in the previous section, the phantom crossing scenario is well explained if the coupling parameter is non-zero but very small positive number compared to unity. In this paper, I have analyzed the model of interacting dark energy using an interesting equation of state which was originally specified in the context of wormhole physics, very similar to the Chaplygin gas that was originally motivated from an engineering problem. The equation of state amalgamates three different forms of dark energy and therefore its inclusion in the interacting model makes the model more comprehensive and unified. From the analysis of this paper, it can be concluded that dark energy could most probably be of the form of generalized phantom energy or generalized Chaplygin gas while phantom energy is ruled out. Also, this work serves as a generalization of earlier studies in this direction [27–29] which dealt with Chaplygin gas only.

References

1. Riess, A.G., et al. (Supernova Search Team Collaboration): *Astron. J.* **116**, 1009 (1998)
2. Perlmutter, S., et al. (Supernova Cosmology Project Collaboration): *Astrophys. J.* **517**, 565 (1999)
3. Bennett, C.L., et al.: *Astrophys. J. Suppl. Ser.* **148**, 175 (2003)
4. Tegmark, M., et al. (Sloan Digital Sky Survey Collaboration): *Phys. Rev. D* **69**, 103501 (2004)
5. Barboza Jr., E.M., et al.: [arXiv:0905.4052](https://arxiv.org/abs/0905.4052)
6. Espana-Bonet, C., Ruiz-Lapuente, P.: *J. Cosmol. Astropart. Phys.* **02**, 018 (2008)
7. Linden, S., Virey, J.-M.: *Phys. Rev. D* **78**, 023526 (2008)
8. Jamil, M., Rashid, M.A.: *Eur. Phys. J. C* **56**, 429 (2008)
9. Chakraborty, W., Debnath, U.: *Phys. Lett. B* **661**, 1 (2008)
10. Usmani, A.A., et al.: *Mon. Not. R. Astron. Soc. Lett.* **386**, L92 (2008)
11. Farrar, G.R., Peebles, P.J.E.: *Astrophys. J.* **604**, 1 (2004)
12. Zimdahl, W., Pavon, D.: *Gen. Relativ. Gravit.* **36**, 1483 (2004)
13. Guo, Z., et al.: *J. Cosmol. Astropart. Phys.* **0505**, 002 (2005)
14. Sussman, R.A., et al.: *Gen. Relativ. Gravit.* **37**, 2117 (2005)
15. Berger, M.S., Shojaei, H.: *Phys. Rev. D* **73**, 083528 (2006)
16. Wang, B., et al.: *Nucl. Phys. B* **778**, 69 (2007)
17. Setare, M.R.: *Phys. Lett. B* **642**, 1 (2006)
18. Sadjadi, H.M.: *J. Cosmol. Astropart. Phys.* **0702**, 026 (2007)
19. Rosenfeld, R.: *Phys. Rev. D* **75**, 083509 (2007)
20. de la Macorra, A.: *Astropart. Phys.* **28**, 196 (2007)
21. Jamil, M., Rashid, M.A.: *Eur. Phys. J. C* **60**, 141 (2009)
22. Setare, M.R., Vagenas, E.C.: *Phys. Lett. B* **666**, 111 (2008)
23. Zhang, H.: [arXiv:0909.3013](https://arxiv.org/abs/0909.3013) [astro-ph.CO]
24. Cruz, N., et al.: [arXiv:0910.1307](https://arxiv.org/abs/0910.1307) [gr-qc]
25. Kuhfittig, P.K.F.: *Gen. Relativ. Gravit.* **41**, 1485 (2009)
26. del Campo, S., et al.: *J. Cosmol. Astropart. Phys.* **0901**, 020 (2009)
27. Zhang, H., Zhu, Z.-H.: *Phys. Rev. D* **73**, 043518 (2006)
28. Wu, P., Yu, H.: *Class. Quantum Gravity* **24**, 4661 (2007)
29. Jamil, M.: *Int. J. Theor. Phys.* doi:[10.1007/s10773-009-0178-8](https://doi.org/10.1007/s10773-009-0178-8)